M.Sc.2nd Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Fluid Mechanics)

Paper MTM – 201

FULL MARKS: 40 : : Time : 02 hours

*Answer any **Four** of the following questions* 4 X 10

1.	(a) What is Newtonian and Non-Newtonian fluids? Discuss with examples. (b) Derive the expression for substantial derivative in Cartesian coordinate systems for the y-velocity component by (i) considering the infinitesimally small fluid element moving in space, and (ii) using the chain rule. Discuss the similarity/dissimilarity among these two derivations. Also discuss the physical significance of substantial derivative.	4+6
2.	(a) Derive the vorticity equation in vector form. (b) Show that $u = \frac{-2xyz}{(x^2+y^2)^2}$, $v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}$ and $w = \frac{y}{(x^2+y^2)}$ satisfies an irrotational motion.	6+4
3.	a) Draw infinitesimally small moving fluid element and show the forces in the y-direction for derivation of the y-component of the Navier-Stokes equation. b) Write the y-component of the Navier-Stokes equations in non-conservation form. Convert this equation to its conservation form. c) With the help of Stokes hypothesis, convert the equation resulted from part-(b) for incompressible flow case into its simplified form.	3+3+4
4.	(a) State and prove Blasius theorem. (b)Show that for an incompressible steady flow with constant viscosity, the velocitycomponents $u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1-\frac{y}{h}\right)$, $v = w = 0$, satisfy the equation of motion, when body force is neglected. h, U, $\left(\frac{dp}{dx}\right)$ are constant and p=p(x)	6+4
5.	(a) Based on the observations of Ludwig Prandtl for boundary layer theory, derive the set of governing equations for the boundary layer flow along a flat plate. (b) Write the physical principals used for the equations of continuity, Navier-Stokes and energy.	7+3

6.	(a) Prove that in the steady motion of a viscous liquid in two dimension, $\gamma\nabla^4\Psi = \frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}$, where (X, Y) isthe impressed force per unit area. (b) What do you mean by boundary layer and boundary layer thickness?	7+3
7.	(a) Write the one-dimensional heat conduction equation. (b) Discretize the above equation using FTCS scheme. What is the truncation error? Also draw the schematic diagram to show the points needed for this discretization. (c) Apply the central difference scheme to the Neumann type boundary condition at the left boundary of the above one-dimensional heat conduction equation. Use the implicit scheme for this discretization at the boundary.	1+5+4
8.	(a) Determine the velocity profile of a steady flow of incompressible fluid between two Porousparallel plate. (b) Discuss the effects of low and high value of Reynold's number on N-S equation.	7+3

M.Sc 2nd Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Numerical Analysis)

Paper MTM – 202

FULL MARKS : 40

Time : 2 hours

Answer any four questions 10x4=40

1.(a) Discuss the stability of second order Runge-Kutta method.

(b) The method $y_{n+1} = y_n + \frac{1}{4}(k_1 + 3k_2), n = 0,1,2,...$ where $k_1 = hf(x_n, y_n)$ and $k_2 = hf(x_n + \frac{2h}{3}, y_n + \frac{2k_1}{3})$ is used to solve the initial value problem $\frac{dy}{dx} = f(x, y) = -10y, y(0) = 1$. Then obtain the step size $\boldsymbol{h}$ for which the method will produce stable results. 3+7

2. (a) Describe Crank-Nicolson implicit method to solve the following head equation: $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$,

subject to the boundary conditions : $u(0, t)=f_1(t)$, $u(1, t)=f_2(t)$, and initial condition $u(x,0) = \phi(x)$.

(b) Explain finite difference method to solve the following IVP.

$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x),\ y(x_0) = a, y'(x_0) = b$, where $P(x)$, $Q(x)$, $R(x)$ are continuous on $[x_0, x_n]$. 5+5

3. (a) Using LU decomposition method, solve the following system of equations:

$$2x + 3y - 6z = 2, \qquad 3x + y + 2z = 7, \qquad -3x + 2y + 6z = 7$$

(b) Use least square method to solve the following system of equations:
$x + 3y = 1,\ x - y = 5,\ -3x + y = 4,\ 3x + 2y = 7$ 5+5

4. (a) Solve the following problem:

$$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}, t > 0, 0 < x < 1$$

The initial conditions are $u(x,0) = f_1(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = f_2(x),\ 0 < x < 1$ and the boundary conditions are $u(0,t) = g_1(t)$ and $u(1,t) = g_2(t), t \geq 0$. 10

(b) Describe the finite difference method to solve the following BVP
$p(x)\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} + r(x)y = s(x), x_0 \leq x \leq x_n,\ y(x_0) = \alpha,\ y(x_n) = \beta.$ 5+5

5. (a) Use fourth order Runge-Kutta method to solve the second order initial value problem $2y''(x) - 6y'(x) + 2y(x) = 4e^x$ with $y(0) = 1$ and $y'(0) = 1$ at $x = 0.2, 0.4$.

(b) Discuss Milne's predictor-corrector formula to find the solution of $y' = f(xy)$, $y(x_0) = y_0$. 6+4

6.(a) Determine whether the following function is spline or not?

$$f(x) = \begin{cases} -x^2 - 2x^3, & x \in [-1, 0] \\ -x^2 + 2x^3, & x \in [0, 1] \end{cases}$$ **2**

(b) Develop the cubic spline of the following information

x:	1	2	3	4
f(x):	1	5	11	8

where $y''(1) = 0 = y''(4)$. Hence compute $y(1.5)$. **6**

(c)What are the basic differences between interpolation and approximation? **2**

7. (a) Explain the ill-conditioned and well- conditioned system. The coefficient matrices of two system of equations are $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ $and B = \begin{pmatrix} 1 & 1 \\ 1 & 1.0001 \end{pmatrix}$. Find the condition of two systems and indicate which system is stable. **5**

(b) For what values of α and β, the quadrature formula

$\int_{-1}^{1} f(x)dx = \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1. **2**

(c) Find the weights w_1, w_2, w_3, so that the relation

$\int_{-1}^{1} f(x)dx = w_1 f(-\sqrt{0.6}) + w_2 f(0) + w_3 f(\sqrt{0.6})$ is exact for the functions $f(x) = 1, x, x^2$. 3

8. (a) Describe power method to find the largest (in magnitude) eigenvalue and the corresponding eigenvector of a matrix.
   (b) Express the polynomial $x^4 + 2x^3 - x^2 + 5x - 9$ in terms of Chebyshev polynomials.

(c) What is the importance of stability analysis of a numerical method to solve a differential equation? 5+2+3

M.Sc.2nd Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Unit-1: Abstract Algebra & Unit-2: Linear Algebra)

Paper MTM – 203

FULL MARKS: 40 : : Time : 02 hours

Unit-I: Abstract Algebra

(*Candidates are required to give their answers in their own words as far as practicable*)

Answer any two questions: 2x10

1.	(a) (i). With proper justification, give an example of an infinite non-commutative solvable group. (ii) Find the class equation of the group S_4. (b) Find the prime ideals and the maximal ideals in the ring $(Z_8, +, .)$.	(2+3)+5
2.	(a) State and proof Isomorphism theorem. (b) Prove that Sl(n,ℝ) is a normal subgroup of GL(n,ℝ). (c) Prove that an abelian group of order 33 is cyclic.	5+3+2
3.	(a) Show that a group of order 1225 is commutative. (b) (i) Are the fields $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ isomorphic? Are they isomorphic as $\mathbb{Q}$-vector spaces? Justify your answer. (ii) Show that $[R : Q]$ is not finite.	5+(3+2)
4.	(a) Prove that every Ideal in $\mathbb{Z}$ is principle Ideal. (b) Show that $\frac{\mathbb{Z}[i]}{(1+3i)} \cong \frac{\mathbb{Z}}{10\mathbb{Z}}$. (c) Prove that any field is an Integral domain.	4+3+3

Unit-II: Linear Algebra

Answer any two questions: 2x10

5.	Answer the questions. a) What is a Quotient space in linear algebra? b) Define linear functional on a vector space with an example. c) Let T be a linear operator on finite-dimensional vector space V. When you say that T is diagonalizable? d) Define Jordan Block with an example or state Spectral theorem on linear algebra. e) Define the characteristic value and characteristic vector of a linear operator on vector space	2 x 5
6.	(a) For two matrices A and B show that AB and BA need not have the same minimal polynomial. (b) Let T be a linear operator on an n-dimensional space V. Then show that The Characteristic and minimal polynomial for T have the same root. (c) Let V and W be vector spaces, and let T: V → W be linear and invertible. Then Show that T^{-1}: W → V is linear.	2+5+3
7.	(a) Let $P(t)$ be a minimal polynomial of a linear operator T on a polynomial on a finite dimensional vector space V. Prove that (i) for any polynomial $g(t)$, if $g(T) = T_0$, then $P(t)$ divides $g(t)$, In particular, $p(t)$ divides the characteristic polynomial of T. (ii) the minimum polynomial of T is unique. b) Let T be a linear operator on $R^3(R)$ which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ 8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Examine that whether T is diagonalizable or not? c) Define linear functional on a vector space. Let T be a linear operator on finite-dimensional vector space V. When you say that T is diagonalizable?	3+4+3
8.	(a) . Let V and W be finite-dimensional vector spaces, and let T: V → W be a linear transformation. Suppose that β is a basis for V. Prove that T is an isomorphism if and only if T(β) is a basis for W (b) Let V be an inner product space, and let T be a linear operator on V. Then T is an orthogonal projection if and only if T has an adjoint $T^*$and $T^{\,2}$= T = T.*	5+5

M.Sc.2nd Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(General Theory of Continuum Mechanics)

Paper MTM – 205

FULL MARKS: 40 : : Time : 02 hours

Answer any four questions: 4 x 10

1.	(a) For velocity field $v_1 = 3x_1^2 x_2, v_2 = 2x_2^2 x_3 and\ v_3 = x_1 x_2 x_3^2$, determine the rate of extension at $P(1,1,1)$ in the direction of $(\frac{3}{5}, 0, \frac{-4}{5})$. Also determine the rate of shear between orthogonal direction $(\frac{3}{5}, 0, \frac{-4}{5})$ and $(\frac{4}{5}, 0, \frac{3}{5})$ (b) Explain volumetric strain.	4+6
2.	Define perfect fluid and prove that the pressure at a point in a perfect fluid has the same magnitude in every direction.	10
3.	(a) If any portion of the moving fluid once becomes irrotational, then show thatit will remain so for all subsequent times provided that the external body forces are conservative and pressure is a function of density alone. (b) State and prove Kelvin's Minimum Energy Theorem.	2+8
4.	Derive the integral of Euler's Equation of motion when body forcesareconservative, pressure is a function of density only and flow is irrotational.	10
5.	(a) Show the equivalence between Eulerian and Lagrangian forms of equationsof continuity. (b) Define principal stress and principal direction of stress. Prove that all principal stresses are real.	5+5
6.	(a) The stress tensor at P is given by $(T_{ij}) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$. Determine the principal stress and principal directions of stress. (b)Find the stress vector at a point on the plane whose normal vector is $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ and state of stress at a point is given by $\begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 0 \\ 3 & 0 & -1 \end{pmatrix}$.	5+5

7.	(a) Derive the condition for a given surface $F(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{t}) = 0$ to be a boundary surface of a fluid motion. (b) Give examples of irrotational and rotational fluid flows.	7+3
8.	(a) Calculate the strain invariants from strain tensor $(E_{ij}) = \begin{pmatrix} 5 & -1 & -1 \\ -1 & 4 & 0 \\ -1 & 0 & 4 \end{pmatrix}$. Determine the principal strains. Obtain strain invariants from them. (b) If the strain tensor at a point is given by $e_{ij} = \begin{bmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{bmatrix}$. Calculate the principal strains and determine the shear between the directions $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$	5+5

M.Sc.2nd Semester examination, 2021

Department of Mathematics, MugberiaGangadharMahavidyalaya

(General Topology)

Paper MTM – 206

FULL MARKS: 20 : : Time : 01 hour

(Candidates are required to give their answers in their own words as far as practicable)

Answer any two questions **2x10**

1.	(a) Is the collection $\tau = \{\mathrm{U}: \mathrm{X} - \mathrm{U} \text{is infinite or empty or all of} \mathrm{X}\}$ a topology on X? (b) If L is a straight-line in the plane, describe the topology L inherits as a subspace of $\mathbb{R}_l \times \mathbb{R}$ and as a subspace of $\mathbb{R}_l \times \mathbb{R}_l$. In each case it is a familiar topology. (c) Show that if X is compact Hausdorff under both $\tau$and τ', then either τ and τ' are equal or they are not comparable.	2+4+4
2.	(a) Give an example of a topological space where a sequence can convergence more than one point. Justify your answer. (b)Let A be a subset of a topological space X. Then show that $x \in \overline{A}$ iff every open set U containing x intersects A. (c) Show that every compact Hausdroff space is normal.	2+4+4
3.	(a) In the finite complement topology on $\mathbb{R}$, to what point or points does the sequence $x_n = \frac{1}{n}$ converge? (b) Show that compactness implies limit point compactness, but not conversely. (c) Show that a closed subspace of a normal space is normal.	3+4+3
4.	(a) Discuss the connectedness of the following sets: (i) $\left\{x sin\left(\frac{1}{x}\right): x \in (0,1)\right\}$ (ii)$\{\lvert x\rvert: x \in (-1,1)\} \cap \{e^x: x \in \mathbb{R}\}$ (b) Give an example of T_1 space which is not T_2 space. (c) Consider the product and box topology on $\mathbb{R}^\omega$. Under what topology thefunction $f: \mathbb{R} \to \mathbb{R}^\omega$ defined by $f(t) = (t, 2t, 3t, \ldots\ldots)$ is continuous ordiscontinuous. Explain your answer.	(2+2)+2+4

M.Sc.2nd Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

Lab. 2: (Language: C- Programming with Numerical Methods)

Paper MTM – 297

FULL MARKS: 25: : Time : 02 hours

Group A

Answer one question **1x10=10**

Q1. Write a program in C to determine the key number from the dynamic sorted list of numbers by an appropriate technique. 10

Q2. Write a program in C to find the key number 25 from the list of sorted numbers {12, 15, 25, 34, 45, 47, 51, 56, 87, 98} using Binary search technique. 10

Q3. Write a program in C to sort a list of names in alphabetical order. 10

Q4. Write a program in C to rewrite the name with surname first followed by initials of first and middle name. Test it for the names: (i) Sunil Kumar Dey (ii) Manas Kumar Mondal (iii) Soma Rani Majhi (iv) Sathi Jana 10

Group B

Answer one question **1x15=15**

Q5. Write a program in C to determine the dominant eigenvalue of a real matrix by the power method. 15

Q6. Write a program in C to find matrix inverse by partial pivoting. Find the inverse of the following matrix A= $\begin{bmatrix} 2 & 4 & 5 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$. 15

Q7. Write a program in C to find the solution of a Tri-diagonal system of equations. 15

Q8. The following table gives pressure of a steam plant at a given temperature. 15

Using Newton's formula, write a program in C to compute the pressure for a temperature of 142°C.

Temperature °C :	140	150	160	170	180
Pressure, kgf/cm^2:	3.685	4.854	6.302	8.076	10.225.